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206. Proposed by W. J. GREENSTREET, A. M., Editor of The Mathematical Gazette, Stroud, England.

$ABCD$ is circumscribed by a circle center O , and it circumscribes a circle radius r . The perpendiculars from C on the sides are x, y, z, u . Show that $\frac{1}{2}AC \cdot BD = r \sum x$.

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

The problem should read "the perpendiculars from O " instead of "the perpendiculars from C ."

Let a, b, c, d denote the sides AB, BC, CD, DA , respectively; x the perpendicular on a ; y , on b ; z , on c ; u , on d ; R —circum-radius.

Then $x = \sqrt{(R^2 - \frac{1}{4}a^2)}$. Now $R = AC/2\sin B$.

$$AC^2 = a^2 + b^2 - 2ab\cos B = c^2 + d^2 + 2cd\cos B.$$

$$\therefore x = \frac{1}{2\sin B} \sqrt{(AC^2 - a^2 \sin^2 B)} = \frac{b - a\cos B}{2\sin B}, \quad y = \frac{a - b\cos B}{2\sin B},$$

$$z = \frac{d + c\cos B}{2\sin B}, \quad u = \frac{c + d\cos B}{2\sin B}. \quad \therefore r \sum x = \frac{r(a+b+c+d) - r(a+b-c-d)\cos B}{2\sin B},$$

$$r = \frac{2\sqrt{(abcd)}}{a+b+c+d}, \quad a+c=b+d, \quad \cos B = \frac{a^2+b^2-c^2-d^2}{2(ab+cd)}.$$

$$\therefore \cos B = \frac{(a-c)(a+c) + (b-d)(b-d)}{2(ab+cd)} = \frac{(a+c)(a+b-c-d)}{2(ab+cd)}.$$

$$\therefore r \sum x = \frac{\sqrt{(abcd)}}{\sin B} - \frac{\sqrt{(abcd)} \cdot (a+b-c-d)^2}{4(ab+cd)\sin B}, \quad \sin B = \frac{2\sqrt{(abcd)}}{ab+cd},$$

$$4(ab+cd)\sin B = 8\sqrt{(abcd)}.$$

$$\therefore r \sum x = \frac{ab+cd}{2} - \frac{(a+b-c-d)^2}{8}. \quad \text{But } d=a+c-d.$$

$$\therefore r \sum x = \frac{ab + c(a+c-b)}{2} - \frac{(b-c)^2}{2} = \frac{ab + ac + bc - b^2}{2}$$

$$= \frac{ac + b(a+c) - b^2}{2} = \frac{ac + bd}{2} = \frac{1}{2}AC \cdot BD.$$

207. Proposed by W. W. HART, University High School, Chicago, Ill.

According to Gauss the circumference of a circle can be divided into n equal parts by ruler and compass only, when n is a prime of the form $2^{2^p} + 1$.

The following construction gives good partial results for n equals any integer. If AB is the diameter of the circle, and C is the vertex of the equilateral triangle ABC , and if D is a point on AB at the distance $2AB/n$ from A , then draw the line CD cutting the circle at E and F ; E being the more remote from

C. $AE=1/n$ circumference approximately. For low values of n this method is very practical; is it practical in general? How great is the error?

I. Solution by H. F. MacNEISH, A. B., Instructor in Mathematics, University High School, Chicago, Ill.

Join OE . Let $\angle ACD=x$ and $\angle AOE=y$; then $DCB=60^\circ-x$; $ADC=120^\circ-x$; $DAE=90^\circ-\frac{1}{2}y$; $AED=30^\circ-x+\frac{1}{2}y$. $AD=2AB/n=4r/n$; $AB=AC=BC=2r$; $AO=OE=r$.

$$\text{In } \triangle ADC: \frac{\sin x}{\sin(120-x)} = \frac{AD}{AC} = \frac{4r/n}{2r} = \frac{2}{n}.$$

$$\therefore \frac{1}{2}n \sin x = \sin(120-x) = \frac{1}{2}\sqrt{3} \cos x + \frac{1}{2}\sin x \dots (1). \quad (n-1)\sin x = \sqrt{3} \sqrt{1-\sin^2 x}.$$

$$\therefore \sin x = \sqrt{\frac{3}{n^2-2n+4}} \dots (2). \quad \therefore \sin(120-x) = \frac{n}{2} \sqrt{\frac{3}{n^2-2n+4}} \dots (3),$$

$$\text{and } \cos(120-x) = \frac{n-4}{2\sqrt{n^2-2n+4}} \dots (4).$$

$$\text{In } \triangle AOE: \frac{AE}{OE} = \frac{\sin y}{\sin(90-\frac{1}{2}y)} = \frac{\sin y}{\cos \frac{1}{2}y}. \quad \therefore AE = \frac{r \sin y}{\cos \frac{1}{2}y} \dots (5).$$

$$\text{In } \triangle AEC: \frac{AE}{AC} = \frac{\sin x}{\sin(30-x+\frac{1}{2}y)}. \quad \therefore AE = \frac{2r \sin x}{\sin(30-x+\frac{1}{2}y)} \dots (6).$$

$$\text{From (5) and (6), } \frac{\sin y}{\cos \frac{1}{2}y} = \frac{2 \sin x}{\sin(30-x+\frac{1}{2}y)}.$$

$$\therefore \sin y [\sin(30-x) \cos \frac{1}{2}y + \sin \frac{1}{2}y \cos(30-x)] = 2 \sin x \cos \frac{1}{2}y,$$

$$\text{or } \sin y [-\cos(120-x) \cos \frac{1}{2}y + \sin \frac{1}{2}y \sin(120-x)] = 2 \sin x \cos \frac{1}{2}y.$$

$$\text{Hence from (1), } \sin y [-\cos(120-x) \cos \frac{1}{2}y + \frac{1}{2}n \sin \frac{1}{2}y \sin x] = 2 \sin x \cos \frac{1}{2}y$$

$$\text{or } \sin x [n \sin y \sin \frac{1}{2}y - 4 \cos \frac{1}{2}y] = 2 \sin y \cos \frac{1}{2}y \cos(120-x)$$

$$\text{or since } \sin y = 2 \sin \frac{1}{2}y \cos \frac{1}{2}y, \sin x [n \sin^2 \frac{1}{2}y - 2] = \sin y \cos(120-x).$$

Then from (2) and (4),

$$\frac{\sqrt{3}}{\sqrt{n^2-2n+4}} [n \sin^2 \frac{1}{2}y - 2] = \frac{n-4}{2\sqrt{n^2-2n+4}} \sin y.$$

$$\therefore 2\sqrt{3} \left[\frac{n(1-\cos y)}{2} - 2 \right] = (n-4) \sin y. \quad \therefore 3(n - n \cos y - 4)^2 = (1 - \cos^2 y)(n-4)^2.$$

$$\therefore \cos y = \frac{(n-4)[3n \pm \sqrt{n^2+16n-32}]}{4(n^2-2n+4)}.$$

Then for the positive value of the radical we obtain the following values of y for $n=3, 4, 5, \dots$

n	$\cos y$	$\log \cos y$	y	$2\pi/n$	Error	Rate of error
3	$-\frac{1}{2}$	—	120°	120°	0	.0000
4	0	—	90°	90°	0	.0000
5	—	9.49107	$71^\circ 57' 12''$	72°	$2' 48''$.0007
6	$\frac{1}{2}$	—	60°	60°	0	.0000
7	—	9.79393	$51^\circ 31' 23''$	$51^\circ 25' 43''$	$5' 40''$.0018
8	—	9.84806	$45^\circ 11' 14''$	45°	$11' 14''$.0042
9	—	9.88248	$40^\circ 16' 38''$	40°	$16' 38''$.0069
10	—	9.90599	$36^\circ 21' 18''$	36°	$21' 18''$.0099
11	—	9.92286	$33^\circ 8' 53''$	$32^\circ 43' 38''$	$25' 15''$.0129
12	—	9.93545	$30^\circ 28' 15''$	30°	$28' 15''$.0157
24	—	9.98363	$15^\circ 38'$	15°	$38'$.0422
48	—	9.99516	$8^\circ 32' 30''$	$7^\circ 30'$	$1^\circ 2' 20''$.1389
90	—	9.99818	$5^\circ 14' 30''$	4°	$1^\circ 14' 30''$.3104
180	—	9.99967	$2^\circ 15'$	2°	$15'$.1250
360	—	9.99995	$49' 40''$ to $54' 40''$	1°	$7' 30''$.1250

The construction therefore has an error of over $1\frac{1}{2}\%$ for values of $n > 12$, and for large values of n the error is very great.

II. Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, West Va.

Let O be the center of the circle, $AO=R=EO$. Then $CO=R\sqrt{3}$, $DO=R-4R/n=(R/n)(n-4)$. \therefore tangent $DCO=(n-4)/n\sqrt{3}$.

$$\sin DCO = \frac{n-4}{\sqrt{(4n^2-8n+16)}}, \quad \cos DCO = \frac{n\sqrt{3}}{\sqrt{(4n^2-8n+16)}},$$

$$\sin CED = \frac{CO \sin DCO}{EO} = \frac{(n-4)\sqrt{3}}{\sqrt{(4n^2-8n+16)}}.$$

$$\sin(90^\circ + DOE) = \sin(180^\circ - DEO - DCO) = \sin(DEO + DCO).$$

$$\therefore \cos DOE = \sin(DEO + DCO) = \frac{(n-4)[3n + \sqrt{(n^2 + 16n - 32)}]}{4n^2 - 8n + 16}.$$

For $n=3, 4$ and 6 the error is nothing.

For $n=5$ the side and angle are a trifle small.

For $n > 6$ the side and angle are too large but the error varies.

For $n=8$, $\cos DOE = .70479$, $DOE = 45^\circ 11' 14.5''$, an error of $11' 14.5''$.

For $n=12$, $\cos DOE = .86186$, $DOE = 30^\circ 28' 25.7''$, an error of $28' 25.7''$.*

For $n=20$, $\cos DOE = .95091$, $DOE = 18^\circ 2' 40''$, an error of only $2' 40''$.

*In solution I, Mr. MacNeish finds the error for $n=12$ to be $28' 15''$, otherwise the two solutions agree. Ed.

For $n=72$, $\cos DOE=.99559$, $DOE=5^\circ 29'$, an error of $29'$.

For large values of n the error is much too great for any purpose.

Also solved by *J. E. SANDERS*, Hackney, Ohio.

CALCULUS.

165. Proposed by CAPT. T. C. DICKSON, Ordnance Department, United States Army, Washington, D. C.

Solve by integration the differential equation

$$\frac{d^2 \xi}{dt^2} + \frac{A}{B} \left(\frac{d\xi}{dt} \right)^2 - \frac{C}{B} = 0,$$

in which A , B , C are given functions of ξ , but independent of t .

Solution by L. E. DICKSON, Ph. D., Assistant Professor of Mathematics, The University of Chicago.

In view of the nature of the coefficients, we regard ξ as the independent variable and t the dependent, the formulae of transformation being

$$\frac{d\xi}{dt} = 1 \div \frac{dt}{d\xi}, \quad \frac{d^2 \xi}{dt^2} = - \frac{d^2 t}{d\xi^2} \div \left(\frac{dt}{d\xi} \right)^3.$$

The given equation thus becomes

$$\frac{d^2 t}{d\xi^2} - \frac{A}{B} \frac{dt}{d\xi} + \frac{C}{B} \left(\frac{dt}{d\xi} \right)^3 = 0.$$

Set $dt/d\xi=y$, whence $t=\int y d\xi$. Then $\frac{dy}{d\xi} - \frac{A}{B}y + \frac{C}{B}y^3 = 0$.

Divide by y^3 and set $z=y^{-2}$. The resulting differential equation

$$\frac{dz}{d\xi} + \frac{2A}{B}z - \frac{2C}{B} = 0$$

is linear. By the usual method, we get

$$z = 2e^{-\lambda} \left(\int \frac{C}{B} e^{\lambda} d\xi + k \right), \quad \lambda \equiv 2 \int \frac{A}{B} d\xi. \quad \therefore t = \int z^{-1/2} d\xi.$$

169. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, O.

Find the value of y from the Eulerian equation

$$y = \int \frac{dx}{(x+\sqrt{3})^2(x^2+1)}.$$